Generalized DEA: an approach for supporting input/output factor determination in DEA

Mohsen Afsharian, Heinz Ahn and Ludmila Neumann
Department of Business Sciences, Technische Universität Braunschweig, Fallersleber-Tor-Wall 23, 38100 Braunschweig, Germany

Purpose – The determination of input and output factors is a well-known source of pitfalls when applying data envelopment analysis (DEA). The purpose of this paper is to contribute to overcome the respective problems of input/output factor determination related to factor selection, dual-role factors and undesirable factors.
Design/methodology/approach – The problems of input/output factor determination are discussed from a goal-oriented perspective, shedding a new light on the role of input/output factors in DEA. This is exemplified by the case of measuring pharmacy stores' efficiency concerning their goal of customer retention.
Findings – The findings suggest to applying a generalized DEA (GDEA). The three steps of this approach include the development of a system of objectives, the derivation of corresponding performance criteria as well as the construction of cost and benefit functions. These functions build the basis for GDEA models, of which one is exemplarily described and applied to the customer retention case.
Research limitations/implications – While traditional DEA implicitly assumes linear cost and benefit functions, GDEA requires to explicitly specifying these functions. In doing so, the approach contributes to solve the problem of factor selection, the problem of dual-role factors and the problem of undesirable factors.
Practical implications – For determining input/output factors in a consistent and transparent manner, it is recommended to apply GDEA in practical benchmarking studies.
Originality/value – GDEA integrates well-known concepts of multi-criteria decision making into traditional DEA. The new approach helps to cope with the challenges of input/output factor determination in DEA.
Keywords – Benchmarking, Data envelopment analysis, Dual-role factors, Factor selection, Generalized DEA, Undesirable factors

The authors thank two anonymous reviewers for their helpful comments and suggestions. Also, the authors gratefully acknowledge the financial support of the Deutsche Forschungsgemeinschaft (DFG) in the context of the research project AH 90/5-1.

https://doi.org/10.24355/dbbs.084-201902081010-0

1. Introduction

Data envelopment analysis (DEA) is known to be a powerful instrument for measuring the relative efficiency of decision making units (DMUs) on the basis of mathematical programming techniques. Since the seminal work of Charnes *et al.* (1978), various DEA models have been developed which mainly differ in the assumptions about the underlying production possibility set and the distance measure to the efficiency frontier. Independently from such differences, the mathematical models use input factors (to be minimized) as well as output factors (to be maximized) as performance criteria and aggregate the respective quantities into an efficiency score for each DMU. One of the major strength of DEA is the endogenous determination of the weights for the aggregation of these inputs/outputs.

However, the application of DEA implies some pitfalls that may affect the validity of the results. An overview of such problems is given by Dyson *et al.* (2001), who also describe approaches to solve them. Some of these approaches are questionable insofar as they do not address the cause but the effect of a problem. For example, the effect that a performance criterion cannot be clearly characterized as *either* input *or* output is caused by different preferences represented by this criterion.

Such preferences sometimes play an underestimated role in DEA application, e.g., when an evaluator has to choose between alternative DEA models. Also, the determination of the relevant performance criteria depends on the preferences of the evaluator. It is agreed with Belton and Stewart (1999), p. 91, who state that "it is impossible to escape value judgements in the building of a DEA model; the selection of inputs and outputs is in itself inherently subjective".

Disregarding the need for subjective choices in applying DEA is a major source of problems. Concerning the determination of input/output factors, three kinds of problems will be addressed in this paper:

- The problem of factor selection: Which input and output factors are relevant in a certain context, which factors are not?
- The problem of dual-role factors: Should certain factors (e.g., bank deposits) be characterized as input factors or as output factors?

https://doi.org/10.24355/dbbs.084-201902081010-0

- The problem of undesirable factors: How should undesirable input factors (e.g., waste determined for a waste-burning power plant) and undesirable output factors (e.g., emissions of such a plant) be taken into account?

It is proposed to approach these problems from the perspective of multi-criteria decision making (MCDM). Especially, the insights concerning the identification of appropriate goals and their operationalization can shed a new light on the role of input/output factors in DEA. This will be explained by discussing the measurement of DEA efficiency for pharmacy stores concerning their goal of customer retention. The findings substantiate the recommendation to expand the traditional DEA by a generalized DEA (GDEA) which originates from Dyckhoff and Allen (2001) in the context of measuring ecological efficiency.

Section 2 illustrates the proposed goal-oriented view on DEA to cope with the outlined problems of input/output factor determination, referring to the pharmacy stores example. Section 3 reviews the GDEA concept and formulates a respective model for the chosen case. Section 4 emphasizes the problem-solving potential of GDEA, addressing three exemplary input/output issues extensively discussed in DEA literature. Section 5 concludes the paper.

2. DEA from a goal-oriented perspective: The case of pharmacy stores' customer retention

2.1. Background

Nowadays, it is essential for pharmacies to complement their core function of providing pharmaceutical care with the retailing business (White and Klinner, 2012, p. 123). Both segments involve managerial challenges. On the one hand, the pharmacies have to compete with a growing number of non-pharmacy retailers. On the other hand, new European legislative regulations for the liberalization of the pharmaceutical market and changes in health care systems as wells as increasingly informed customers require essential modifications in operating pharmacies to be competitive (Feletto *et al.*, 2010, p. 164). This pressure has stimulated a search for appropriate managerial tools that enable to evaluate the performance of pharmacies in the sense of competing DMUs.

https://doi.org/10.24355/dbbs.084-201902081010-0

A common approach to measure the wide range of pharmacies' activities is the use of multiple key performance indicators (KPIs) for ratio analyses (Barnum *et al.*, 2011, p. 60). The problem is that “pharmacies performing well on some ratios usually perform poorly on others” (Schumock *et al.*, 2009, p. 1660). In such a context, it is challenging to get an aggregated impression of a DMU's achieved performance and to compare it with other DMUs. DEA provides a solution to this problem, calculating a single measure of efficiency from a given set of inputs and outputs.

Here, it is abstained from repeating the basics and advantages of DEA which are described in numerous publications (see, e.g., Thanassoulis, 2001; Rickards, 2003). Instead, three problems of input/output factor determination are investigated. These problems can occur in any DEA application, like in the project with a European pharmacy chain which is addressed in the following. When considering DEA to compare the stores of this chain, the following questions arose:

- Problem of factor selection (Wagner and Shimshak, 2007): What are the appropriate performance criteria?
  The pharmacy chain provided a multitude of KPIs. Examples for input factors of the stores' service processes are worked hours and store square meters. Examples for outputs are the number of customers purchasing over the counter products (available without prescription), the number of customers buying drugs (available only with prescription) and the total number of prescriptions. However, DEA itself is not able to clarify which of these criteria are suitable to reflect the overall performance of the stores or, e.g., customer retention as a particular aspect of their performance. Only the evaluators can select appropriate criteria, depending on how they specify performance.

- Problem of dual-role factors (Cook *et al.*, 2006): How to deal with performance criteria which are not clearly input or output factors?
  Subject to the examined segment of a multi-stage production process, performance criteria can either be characterized as input or output factors. Beyond this widely discussed but still present DEA pitfall, it is typical for service processes that the respective service can neither be described as input nor as output. Regarding the example of customer retention of the pharmacy stores, e.g., health counselling

https://doi.org/10.24355/dbbs.084-201902081010-0

services provided by the employees to the customers embody both input and output aspects in uno actu.

- Problem of undesirable factors (Seiford and Zhu, 2002): How to deal with inputs to be maximized and outputs to be minimized?

  In order to measure the stores' efficiency concerning customer retention, several undesirable factors are worth considering. For instance, the telephone hotline hours for health counselling services can be regarded as an input to be maximized, while the number of subsequent deliveries of drugs (which were not available in the store when the customers wanted to purchase them) can be regarded as an output to be minimized. The DEA literature only provides ad hoc protocols to cope with this pitfall. This is problematic from an application point of view, as the selection among these protocols can have a strong effect on the efficiency scores for the DMUs under evaluation.

2.2. Goal-oriented approach to determine performance criteria for DEA

In order to shed another light on the problems addressed above, performance is defined as the fulfillment of goals pursued. This point of view suggests a procedure for performance measurement which originates from MCDM: First of all, there is a need to clarify what the relevant goals are; hereinafter, it has to be specified how these goals can be operationalized by suitable performance criteria in order to quantify the achieved performance level (see also Agrell and Steuer, 2000). The selection of such criteria is oriented towards their ability to adequately reflect the respective goal level. In contrast, it is not important whether these performance criteria can be characterized as input or output factors of a production process.

As a consequence, DEA has to be seen in a broader context, supporting the initial proposal of Dyckhoff and Allen (2001), explicated by Dyckhoff and Ahn (2010), for a generalized DEA (GDEA). Their GDEA models are designed to measure efficiency with regard to goals to be minimized versus goals to be maximized. Input/output factors are only relevant to the extent that they serve for quantifying the goals. The present paper aims to show that this goal-oriented approach is the key to cope with the three aforementioned pitfalls of determining input/output factors in DEA.

https://doi.org/10.24355/dbbs.084-201902081010-0

For an exemplary elucidation, the case of the European pharmacy chain is used. It is assumed that the central management will compare the efficiency of chain's stores with regard to customer retention. The (hypothetical) results of a goal-oriented process to specifying appropriate performance criteria are depicted in Figure 1. With the help of this example, the problem solving potential of the approach will now be described. It should be noted that the focus is on procedural aspects to generate the results, not on the results themselves, because the latter depend to a large extent on preferences of the evaluator(s).

Figure 1. Determination of performance criteria for DEA from a goal-oriented perspective: The example of customer retention

Customer retention can be considered as one of the goals of a pharmacy chain's balanced scorecard (see, e.g., Kaplan and Norton, 2003; Shutt, 2003). To measure the efficiency with which such a fundamental goal is achieved, a respective set of performance criteria and their functional relationships can be determined according to the following three steps:

Step 1: Development of a system of objectives

In a first step, a simple but comprehensive and at the same time redundance-free system of lower-level goals has to be derived. The term 'system' indicates here that it is recommended to structure the lower-level goals – hereinafter called objectives. In the context of DEA, it is meaningful to divide them into s objectives to be minimized (g = 1,...,s, with achieved values k_g) and r objectives to be maximized (h = 1,...,r, with achieved values l_h), subsequently called *cost* and *benefit objectives*, respectively. Under consideration of multipliers v_g and u_h, the goal-oriented (GDEA) efficiency of a DMU can then be defined as (Dyckhoff and Ahn, 2010, p. 1261):

$$\Theta := \frac{L}{K} = \frac{\sum_{h=1}^{r} u_h l_h}{\sum_{g=1}^{s} v_g k_g} \qquad (1)$$

Referring to the example in Figure 1, the group of cost objectives comprises the *cost of service* for the customers, the customers' *waiting time in the store* and their *waiting time for drugs not in stock*. Concerning the benefit objectives, it is distinguished between the *customer advisory service concerning drugs* and the *customer advisory service*

https://doi.org/10.24355/dbbs.084-201902081010-0

concerning OTC (over the counter) *sales*, provided by the service personnel of the pharmacy stores. It is emphasized once more that this set of objectives is just an example of how to measure customer retention.

The MCDM literature provides some basic concepts to facilitate the development of such a system (Keeney and Raiffa, 1993, Chapter 2; Eisenführ *et al.*, 2010, Chapter 3). Especially, it is recommended to build a hierarchy of objectives, following a set of rules; e.g., breaking down a goal or objective into its constituent parts helps avoid redundancy between the lower-level objectives and to ensure that all relevant aspects are covered. Although results will always be influenced by individual preferences, the structured procedure to derive a system of objectives helps third parties to understand and evaluate these results.

Step 2: Derivation of suitable performance criteria

The second step refers to the need to determine performance criteria which allow to quantifying the achieved values k_g and l_h of the identified objectives. In the MCDM literature, these performance criteria are often called attributes, whereby natural, proxy and artificial ones are distinguished (Eisenführ *et al.*, 2010, pp. 72–73). Table I describes these kinds of attributes and their occurrence in the example.

Table I. Characterization of attributes as performance criteria to measure the achievement of objectives

Most attributes in the example can be classified as input or output factors of the business processes in a pharmacy store. However, such an unambiguous characterization is not possible *for the number of health counselling services in the store*. As already mentioned, this attribute refers to a service process which typically encompasses at the same time input and output properties. The same would apply to *telephone hotline hours for health counselling services*, if this attribute is meant in the sense of the actual counselling time, not the stand-by time.

In any case, the classification of the attributes as input/output factors is not crucial when measuring goal-oriented efficiency. The reason is that the MCDM approach to DEA efficiency measurement abstains from splitting up a production (or service) process into an input, throughput and output phase. In contrast, the traditional DEA is based on such a three-phase point of view. However, this is not a methodical advantage as long as the throughput phase is "regarded as a black box" (Rousseau and Rousseau, 1997).

https://doi.org/10.24355/dbbs.084-201902081010-0

Step 3: *Construction of cost and benefit functions*

The qualitative relation between objectives and attributes has to be converted into value functions (i.e. cost functions and benefit functions) which describe the quantitative links between them. In the best case, objectives are described by natural attributes, which means that the respective value functions are obvious. For instance, it will be generally agreed that the relation between the objective *cost of service* (k_1) and its attributes *cost of service personnel* (CSP) and *cost of expired drugs* (CED) should be described by the linear cost function k_1 = CSP + CED.

In the case of a proxy attribute, the respective value function is subject to an individual estimation of the quantitative relation between the objective and its attribute. Referring to k_3 = SDD in Table 1, e.g., the *waiting time for drugs not in stock* (k_3) is modeled as a cost linear function of the attribute *number of subsequent drug deliveries* (SDD). The latter can be characterized as an output to be minimized. Unlike in traditional DEA, this undesirable factor does not cause any methodical problems in the goal-oriented DEA approach.

Artificial attributes are a special case in the sense that they measure an objective by a function of at least two different performance criteria. In the example, the *customer advisory service concerning drugs* (l_1) is quantified by a linear benefit function of the *telephone hotline hours for health counselling services* ($\mathrm{THH}_{\mathrm{norm}}$) and the *number of health counselling services in the store* ($\mathrm{NHS}_{\mathrm{norm}}$); thereby, normalized values of these two attributes are suggested to reduce scale size effects on the results: $l_1 = \mathrm{THH}_{\mathrm{norm}} + \mathrm{NHS}_{\mathrm{norm}}$.

Any normalization should be based on reasonable attribute ranges. For the *telephone hotline hours for health counselling services*, e.g., a range of [0, 720] seems to be appropriate with respect to a time horizon of one month with 30 days and a single hotline which can be operated between zero and (30 days • 24 hours/day =) 720 hours. If such natural limits do not exist, a realistic attribute range has to be estimated. Alternatively, the range resulting from the given data set may be considered. However, this will cause range size effects when additional DMUs with extreme attribute values are subsequently included into the analysis.

https://doi.org/10.24355/dbbs.084-201902081010-0

The construction of the exemplarily described cost and benefit functions – in Figure 1 labeled as GDEA level – is a prerequisite to apply GDEA. After depicting the basics of this concept in the following, the GDEA models of type BCC (see Banker *et al.*, 1984) are introduced and specified for the considered example of customer retention.

3 Measuring goal-oriented efficiency of customer retention by applying GDEA

3.1 The GDEA concept

GDEA has its origin in Dyckhoff and Allen (2001) who suggested a framework of how to measure ecological efficiency on the basis of a comprehensive preference structure. For this special purpose, they have proposed "a generalization of basic DEA models … by incorporating a multi-dimensional value function $\boldsymbol{f}$" (p. 312), whereby ζ ($\zeta = 1,\ldots,Z$) quotes the number of objectives considered. Each objective is described as a function of inputs x_i ($i = 1,\ldots,m$) and outputs y_j ($j = 1,\ldots,n$) as follows (p. 320):

$$\boldsymbol{f}: \quad \mathrm{IR}^{m+n} \rightarrow \mathrm{IR}^{Z}$$

$$\boldsymbol{f}\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix} = \begin{pmatrix} f_1\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix} \\ \vdots \\ f_\zeta\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix}\end{pmatrix} \quad (\zeta = 1,\ldots,Z) \tag{2}$$

Dyckhoff and Ahn (2010) explicate that this approach is not only applicable for ecological efficiency, but for any form of functional efficiency, referring to the already introduced division of a set of performance criteria into $g = 1,\ldots,s$ cost objectives (to be minimized) and $h = 1,\ldots,r$ benefit objectives (to be maximized). The values k_g and l_h of the $s + r = Z$ objectives can be measured according to (2) by non-negative value functions $k_g = f_g$ $(\mathbf{x}, \mathbf{y})$ for the cost objectives and non-negative value functions $l_h = f_{s+h}$ $(\mathbf{x}, \mathbf{y})$ for the benefit objectives. Together, these functions represent the multi-dimensional value function $\boldsymbol{f}$ defined in (2).

Of particular importance is the case that $\boldsymbol{f}$ as well as the objective functions are linear: The value k of a cost objective g is equal to the linear combination of all undesirable

https://doi.org/10.24355/dbbs.084-201902081010-0

input and output factors τ contributing to g with input/output weights $c_{g,\tau}$. Analogously, the value l of a benefit objective h is equal to the linear combination of all desirable input and output factors τ contributing to h with weights $e_{h,\tau}$ (Dyckhoff and Ahn, 2010, p. 1264):

$$k_g(\mathbf{x},\mathbf{y}) = \sum_{\tau=1}^{m} c_{g,\tau} x_\tau + \sum_{\tau=1}^{n} c_{g,m+\tau} y_\tau$$

and (3)

$$l_h(\mathbf{x},\mathbf{y}) = \sum_{\tau=1}^{m} e_{h,\tau} x_\tau + \sum_{\tau=1}^{n} e_{h,m+\tau} y_\tau$$

The functional relations in (3) build the basis for the specification of GDEA models. Starting point is the definition of goal-oriented (GDEA) efficiency according to (1), to which the Charnes/Cooper-transformation can be applied. Dyckhoff and Ahn (2010, p. 1264) demonstrate this for the case that the improvement of the cost objectives is emphasized, under the assumption of constant returns to scale. By analogy with the CCR-I model of traditional DEA (Charnes *et al.*, 1978), the resulting model can be named as GCCR-MIN model, where G stands for generalized and MIN for focusing on the objectives to be minimized.

3.2 Introduction of the GBCC-MAX model

With regard to the example of the European pharmacy chain, it may be assumed that the pharmacy stores – as the π DMUs under consideration (ρ = 1,…,π) – operate under variable returns to scale. For this scenario, a GBBC-MIN and a GBBC-MAX model (labeled analogously to the traditional BCC-I and BCC-O model of Banker *et al.*, 1984) can be developed. Regarding the respective DMU^o under evaluation, the former model has the multiplier form

$$\begin{aligned}
&\theta_*^o = \max L^o = \sum_{h=1}^{r} u_h l_h^o - \varpi^o \\
\text{s.t.}\quad &\sum_{g=1}^{s} v_g k_g^o = 1 \\
&\sum_{h=1}^{r} u_h l_h^\rho - \sum_{g=1}^{s} v_g k_g^\rho - \varpi^o \le 0, \qquad \rho = 1,\ldots,\pi \\
&v_g \ge 0,\ g = 1,\ldots,s; \qquad u_h \ge 0,\ h = 1,\ldots,r \\
&\varpi^o \ \textit{free in sign}
\end{aligned} \tag{4}$$

https://doi.org/10.24355/dbbs.084-201902081010-0

and the envelopment form

$$
\begin{aligned}
&\theta_*^o = \min\ \theta^o \\
\text{s.t.}\quad &\sum_{\rho=1}^{\pi} \lambda^\rho l_h^\rho - l_h^o \geq 0, \qquad h = 1,\dots,r \\
&\sum_{\rho=1}^{\pi} \lambda^\rho k_g^\rho - \theta^o k_g^o \leq 0, \qquad g = 1,\dots,s \\
&\sum_{\rho=1}^{\pi} \lambda^\rho = 1 \\
&\lambda^\rho \geq 0, \qquad \rho = 1,\dots,\pi \\
&\theta^o \ \textit{free in sign}
\end{aligned} \tag{5}
$$

The GBBC-MAX model has the multiplier form

$$
\begin{aligned}
&\eta_*^o = \min K^o = \sum_{g=1}^{s} v_g k_g^o - \varpi^o \\
\text{s.t.}\quad &\sum_{h=1}^{r} u_h l_h^o = 1 \\
&\sum_{h=1}^{r} u_h l_h^\rho - \sum_{g=1}^{s} v_g k_g^\rho + \varpi^o \leq 0, \qquad \rho = 1,\dots,\pi \\
&v_g \geq 0,\ g = 1,\dots,s; \qquad u_h \geq 0,\ h = 1,\dots,r \\
&\varpi^o \ \textit{free in sign}
\end{aligned} \tag{6}
$$

and the envelopment form

$$
\begin{aligned}
&\eta_*^o = \max \eta^o \\
\text{s.t.}\quad &\sum_{\rho=1}^{\pi} \lambda^\rho l_h^\rho - \eta^o l_h^o \geq 0, \qquad h = 1,\dots,r \\
&\sum_{\rho=1}^{\pi} \lambda^\rho k_g^\rho - k_g^o \leq 0, \qquad g = 1,\dots,s \\
&\sum_{\rho=1}^{\pi} \lambda^\rho = 1 \\
&\lambda^\rho \geq 0, \qquad \rho = 1,\dots,\pi \\
&\eta^o \ \textit{free in sign}
\end{aligned} \tag{7}
$$

In the following, the GBBC-MAX model is further specified, assuming linear cost and benefit functions. Inserting (3) into (6) and (7), the resulting model has the multiplier form

https://doi.org/10.24355/dbbs.084-201902081010-0

$$\eta_*^o = \min K^o = \sum_{\tau=1}^{m} \tilde{v}_\tau x_\tau^o + \sum_{\tau=1}^{n} \tilde{v}_{m+\tau} y_\tau^o - \varpi^o \tag{8}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{m} \tilde{u}_\tau x_\tau^o + \sum_{\tau=1}^{n} \tilde{u}_{m+\tau} y_\tau^o = 1$$

$$\sum_{\tau=1}^{m} \left(\tilde{u}_\tau - \tilde{v}_\tau\right) x_\tau^\rho + \sum_{\tau=1}^{n} \left(\tilde{u}_{m+\tau} - \tilde{v}_{m+\tau}\right) y_\tau^\rho + \varpi^o \le 0, \qquad \rho = 1,\dots,\pi$$

$$\tilde{v}_\tau = \sum_{g=1}^{s} v_g c_{g,\tau}, \quad \tilde{u}_\tau = \sum_{h=1}^{r} u_h e_{h,\tau}, \qquad \tau = 1,\dots,m+n$$

$$v_g \ge 0,\ g = 1,\dots,s; \qquad u_h \ge 0,\ h = 1,\dots,r$$

$$\varpi^o \textit{ free in sign}$$

and the envelopment form

$$\eta_*^o = \max \eta^o \tag{9}$$

$$\text{s.t.} \quad \sum_{\tau=1}^{m} e_{h,\tau} \left(\tilde{x}_\tau - \eta^o x_\tau^o\right) + \sum_{\tau=1}^{n} e_{h,m+\tau} \left(\tilde{y}_\tau - \eta^o y_\tau^o\right) \ge 0, \qquad h = 1,\dots,r$$

$$\sum_{\tau=1}^{m} c_{g,\tau} \left(\tilde{x}_\tau - x_\tau^o\right) + \sum_{\tau=1}^{n} c_{g,m+\tau} \left(\tilde{y}_\tau - y_\tau^o\right) \le 0, \qquad g = 1,\dots,s$$

$$\tilde{x}_\tau = \sum_{\rho=1}^{\pi} \lambda^\rho x_\tau^\rho, \qquad \tau = 1,\dots,m$$

$$\tilde{y}_\tau = \sum_{\rho=1}^{\pi} \lambda^\rho y_\tau^\rho, \qquad \tau = 1,\dots,n$$

$$\sum_{\rho=1}^{\pi} \lambda^\rho = 1$$

$$\lambda^\rho \ge 0, \qquad \rho = 1,\dots,\pi$$

$$\eta^o \textit{ free in sign}$$

In the very particular case **k** = **x** and **l** = **y** of linear functions (3), the GDEA models are identical to the respective well-known traditional DEA models. Concerning the envelopment form (9), the generalization is reflected by the restrictions for the cost objectives $h = 1,...,r$ and the benefit objectives $g = 1,...,s$. Concerning the multiplier form (8), new multiplier variables $\tilde{v}_\tau$ and $\tilde{u}_\tau$ for the inputs and outputs are introduced, which result from the multipliers for both kind of objectives. $\tilde{v}_\tau$ can be interpreted as a cost multiplier, resulting from the sum of the products of the multiplier of the cost

https://doi.org/10.24355/dbbs.084-201902081010-0

objective and the corresponding input/output weights for each cost objective. Analogously, ϑ_{τ}^{o} is a corresponding benefit (e.g. revenue) multiplier.

3.3 The GBCC-MAX model for measuring the efficiency of customer retention

The introduced models provide the theoretical framework for measuring goal-oriented GDEA efficiency. On the basis of the considerations presented in Chapter 2 and summarized in Figure 1, this framework can now be tailored to the case of measuring the pharmacy stores' efficiency of customer retention. The respective GBCC-MAX model in the multiplier form reads:

$$\eta_*^o = \min K^o = v_1 k_1^o + v_2 k_2^o + v_3 k_3^o - \varpi^o \tag{10}$$

$$\begin{aligned}
\text{s.t.}\quad & u_1 l_1^o + u_2 l_2^o = 1 \\
& u_1 l_1^\rho + u_2 l_2^\rho - v_1 k_1^\rho - v_2 k_2^\rho - v_3 k_3^\rho + \varpi^o \le 0, \qquad \rho = 1,\ldots,\pi \\
& v_g \ge 0,\ g = 1,2,3; \qquad u_h \ge 0,\ h = 1,2 \\
& \varpi^o \textit{ free in sign}
\end{aligned}$$

or, with respect to (3),

$$\eta_*^o = \min K^o = v_1\left(\mathrm{CSP}^o + \mathrm{CED}^o\right) + v_2\mathrm{CWT}^o + v_3\mathrm{SDD}^o - \varpi^o \tag{11}$$

$$\begin{aligned}
\text{s.t.}\quad & u_1\left(\mathrm{THH}_{\mathrm{norm}}^o + \mathrm{NHS}_{\mathrm{norm}}^o\right) + u_2\mathrm{OTC}^o = 1 \\
& u_1\left(\mathrm{THH}_{\mathrm{norm}}^\rho + \mathrm{NHS}_{\mathrm{norm}}^\rho\right) + u_2\mathrm{OTC}^\rho - v_1\left(\mathrm{CSP}^\rho + \mathrm{CED}^\rho\right) - v_2\mathrm{CWT}^\rho - v_3\mathrm{SDD}^\rho + \varpi^o \le 0, \\
& \rho = 1,\ldots,\pi \\
& v_g \ge 0,\ g = 1,2,3; \qquad u_h \ge 0,\ h = 1,2 \\
& \varpi^o \textit{ free in sign}
\end{aligned}$$

While it is distinguished between inputs and outputs in (3), this distinction is not necessary to formulate (11). Especially, the $\mathrm{THH}_{\mathrm{norm}}$ criterion (normalized telephone hotline hours for health counselling services) can be taken into account in the model straightforwardly, although its characterization as input or output is not unequivocally possible. This issue will be further discussed in the concluding section.

The envelopment form of the GBCC-MAX model reads:

$$
\begin{aligned}
& \eta_*^o = \max \eta^o \\
\text{s.t.} \quad & \sum_{\rho=1}^{\pi} \lambda^\rho l_1^\rho - \eta^o l_1^o \geq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho l_2^\rho - \eta^o l_2^o \geq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho k_1^\rho - k_1^o \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho k_2^\rho - k_2^o \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho k_3^\rho - k_3^o \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho = 1 \\
& \lambda^\rho \geq 0, \qquad \rho = 1,\ldots,\pi \\
& \eta^o \textit{ free in sign}
\end{aligned}
\tag{12}
$$

or, with respect to (3),

$$
\begin{aligned}
& \eta_*^o = \max \eta^o \\
\text{s.t.} \quad & \sum_{\rho=1}^{\pi} \lambda^\rho \left(\mathrm{THH}_{\mathrm{norm}}^\rho + \mathrm{NHS}_{\mathrm{norm}}^\rho \right) - \eta^o \left(\mathrm{THH}_{\mathrm{norm}}^o + \mathrm{NHS}_{\mathrm{norm}}^o \right) \geq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho \mathrm{OTC}^\rho - \eta^o \mathrm{OTC}^o \geq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho \left(\mathrm{CSP}^\rho + \mathrm{CED}^\rho \right) - \left(\mathrm{CSP}^o + \mathrm{CED}^o \right) \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho \mathrm{CWT}^\rho - \mathrm{CWT}^o \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho \mathrm{SDD}^\rho - \mathrm{SDD}^o \leq 0 \\
& \sum_{\rho=1}^{\pi} \lambda^\rho = 1 \\
& \lambda^\rho \geq 0, \qquad \rho = 1,\ldots,\pi \\
& \eta^o \textit{ free in sign}
\end{aligned}
\tag{13}
$$

The models (10) to (13) provide an efficiency score for each DMU which can be referred to as GDEA efficiency of customer retention. The respective explications illustrate how GDEA broadens the traditional DEA approach by combining its focus on performance criteria with the goal-oriented focus of MCDM.

As the lower-level goals derived for measuring the fundamental goal of customer retention represent a hypothetical example, a comparison between respective DEA and GDEA results on the basis of numerical data is not useful. Nevertheless, this example reveals the cause of the three addressed problems of determining input/output factors in

https://doi.org/10.24355/dbbs.084-201902081010-0

DEA: It is decisive that DEA is just a special case of GDEA. While GDEA requires to explicitly specify cost and benefit functions, this is only implicitly done in traditional DEA, with the very strict assumption of linear functions in the form of $k_g = x_g$ and $l_h = y_h$ ($g = 1,...,s$ and $h = 1,...,r$).

Refraining from this assumption, the GDEA approach can contribute to cope with the challenges of input/output factor determination and shed a new light on important issues discussed in DEA literature. In the following, three of these issues are exemplarily addressed.

4 Problems of input/output factor determination in the light of the GDEA approach

4.1 Factor selection and the issue of measuring bank efficiency

The relevance of input/output factors depends on the objectives to be pursued by the DMUs. Although this may seem obvious, many DEA applications give rise to suspect that data are preliminary not chosen with regard to their appropriateness but due to their availability. In contrast, a sound DEA application requires first to systematically deriving the objectives to be taken into account. This is a prerequisite for the next step of selecting reasonable performance factors.

Only in a few DEA research areas, the topic of factor selection has been profoundly discussed. One prominent example is the area of bank efficiency measurement. Here, so-called bank behavior models are used as a basis to derive specific sets of input/output factors for DEA applications. The two mainly applied behavior models are the intermediation approach and the production approach. While the latter focuses on the service producing role of banks, the former focuses on their fund intermediating role. However, the respective literature does not explicitly refer to the underlying sets of goals and lower-level objectives. Even from a certain behavior perspective, these sets can vary from case to case. For example, commercial banks have a different business model than savings banks, so their performance must be reflected by – at least partly – different goals and objectives (see Ahn and Le, 2014).

It does therefore not astonish that comprehensive reviews, e.g., by Fethi and Pasiouras (2010) as well as Paradi and Zhu (2013), conclude that there has been so far no broad

https://doi.org/10.24355/dbbs.084-201902081010-0

consensus about performance criteria for DEA-based bank efficiency measurement. The GDEA approach reveals that such a consensus is not possible, since these criteria have to be individually determined from the perspective of the evaluators. These evaluators are typically not the (G)DEA researchers themselves but the ones who will adjust their decision making according to the results of the efficiency analysis and take measures to enhance the DMUs' performance.

To provide instrumental support, DEA guidelines like the ones of Golany and Roll (1989) as well as of Emrouznejad and De Witte (2010) should be complemented by a phase which comprises the three steps described in Section 2.2 to apply the GDEA approach. Such a phase will not only help the evaluators to solve the factor selection problem, but also enables third parties to understand the underlying assumptions. To this end, MCDM concepts for determining, structuring and operationalizing cost and benefit objectives can be made fruitful.

4.2 Dual-role factors and the issue of third-party funds

While a natural attribute arises unambiguously from the objective under consideration, proxy and artificial attributes leave scope for the DEA user how to choose and construct them, respectively. In traditional DEA, this leads to the problem of dual-role factors, which has also been discussed under the heading of flexible measures (Cook and Seiford, 2009, Section 5.6). Such measures are associated with both desired and undesired effects. In this context, the question is raised in DEA literature when a certain dual-role factor should be characterized as input and when it should be characterized as output. From the GDEA point of view, this question is misleading. It is not the classification of the factors that matters, but their potential to accurately quantify the relevant cost and benefit objectives. It is possible that a particular factor is appropriate to be used in measuring several – perhaps opposing – objectives.

Third-party funds in the higher education sector are a corresponding example (for bank deposits as another example, see, e.g., Fethi and Pasiouras, 2010). On the one hand, they are part of the financial resources used to produce research output. These financial resources represent an objective to be minimized, which can be measured by natural attributes that quantify certain shares of the money spent. To this respect, third-party funds can unambiguously quantify that share of expenditures for research which is not

https://doi.org/10.24355/dbbs.084-201902081010-0

financed by the regular budget of the university but by, e.g., companies or governmental funding organizations (see, e.g., Johnes and Johnes, 1995).

On the other hand, third-party funds may also be applied as an indicator to measure the research output as an objective to be maximized (see, e.g., Başkaya and Klumpp, (2014); this can be justified if there is no better indicator. In such cases, the funds granted to researchers or research institutions are assumed to capture their ability of creating new knowledge. Thereby, the nature of the third-party funds as a performance measure changes. While they are a natural attribute (to be minimized) for part of the financial resources, they are a proxy attribute (to be maximized) for research output (see Keeney, 1992, pp. 101–103).

From the point of view of MCDM, this ambiguity of third-party funds is unproblematic, since it reflects the present situation of competing objectives. Accordingly, it is not important for a GDEA calculation whether funds are labeled as an input, output, or even intermediate good of the research process (Fandel, 2007). The crucial aspect is, however, to what extent certain performance criteria – here funds – can quantify the objectives pursed by the evaluators. This aspect is mostly neglected in traditional DEA literature, leading to efficiency analyses with often questionable sets of performance criteria.

4.3 Undesirable factors and the issue of airport performance

In analogy to dual-role factors, the problem of undesirable factors occurs only if proxy or artificial attributes are used. Taking the pharmacy store example, the natural attribute to measure the *waiting time in the store* (as objective k to be minimized) would be the actual waiting time itself. If corresponding data is not available, the *number of complaints about waiting time in the store* (CWT) could be considered as a proxy attribute. In traditional DEA, the problem would arise that this attribute is an output to be minimized. This problem can be solved in GDEA by applying the cost function $k = \text{CWT}$.

In order to discuss this aspect in a more general way, let us refer to the practical example of measuring airport performance. Recent DEA literature on this subject focuses on undesirable outputs of airport processes and mathematical approaches to incorporate such factors into DEA models (for an overview of these approaches, see

https://doi.org/10.24355/dbbs.084-201902081010-0

Ramli and Munisamy, 2013). Besides noise affecting the communities around airports (Yu *et al*., 2008), flight delays are especially emphasized (see, e.g., Lozano *et al*., 2013, and Pathomsiri *et al*., 2008).

In the last-mentioned paper, flight delays are taken into account by two outputs, namely the number of delayed flights (criterion 1) and their time delays in minutes (criterion 2), "in order to capture more completely the effect of delays" (Pathomsiri *et al*., 2008, p. 241). However, it has been shown that a greater number of performance criteria can reduce the discriminating power of DEA results. The question therefore arises whether it is justifiable to use two criteria instead of one to model flight delays.

This issue can be investigated from different perspectives. From a MCDM-oriented view, the undesirable outputs chosen by Pathomsiri *et al*. (2008) can be characterized as two redundant attributes to measure delays: while criterion 1 is an unweighted aggregation of delayed flights, criterion 2 can be seen as an aggregation of such flights weighted by the minutes of their delay. Hence, criterion 2 includes criterion 1, which therefore should be excluded from an efficiency analysis in the context of (G)DEA.

In MCDM, the problem of redundant criteria is also discussed under the topic of the goal splitting bias (see, e.g., Eisenführ et al., 2010, p. 155 and p. 386): Measuring a specific objective with more than one attribute leads to the phenomenon that evaluators tend to overweight the respective attribute. A DEA-based analysis is not immune to this effect, since splitting up a main criterion into different sub-criteria allows a DEA model to re-allocate weights in favour of the main criterion. The GDEA approach makes it easier to be aware of such pitfalls; in this respect, a visualization of the relations between the goal level – with its objectives to be minimized or maximized – and the performance criteria level will be helpful, as has been exemplified in Figure 1.

https://doi.org/10.24355/dbbs.084-201902081010-0

5 Final conclusions

To sum up, the goal-oriented perspective of GDEA provides plausible arguments to consider this approach for benchmarking the efficiency of DMUs. It can improve the operationalization of performance by demanding to reflect about the pursued fundamental goal(s), the respective lower-level objectives, and suitable cost and benefit functions. This makes it necessary for researchers – especially in practical applications – to strongly involve the evaluators as those who will finally make decisions based on the results of the efficiency analysis. At the same time, subjective components of the evaluation process will become more apparent than in the case of the traditional DEA approach. Another consequence can be that important aspects of performance gain attention which have been neglected so far. In practice, however, such effects can be both welcomed and unwelcomed.

From an academic perspective, the GDEA approach raises a series of questions which are interesting for future research. For example, the development of non-oriented GDEA models may be necessary for certain cases. Also, supplemental analyses which are typical for traditional DEA applications, e.g., correlation studies, should be reflected in the light of GDEA. With a more practical focus, it may be worth considering multi-period situations, especially under changing conditions (see, e.g., Afsharian and Ahn, 2014); in such situations, short-term and long-term goals have to be simultaneously taken into account. Likewise, to study behavioral issues of using GDEA in practice promises manifold insights.

A further broad field of research is the comparison of GDEA with other methodologies of input/output factor selection. Of special interest appear approaches referring to expert knowledge. For example, Edirisinghe and Zhang (2007, p. 1670) propose that "input/output categorization is endogenously determined by a model that seeks the highest correlation between stock returns and efficiency metric." This is a "generalized DEA model" in so far as the inputs and outputs are not specified a priori, while our generalized DEA approach is more comprehensive, focusing also on cost and benefit objectives as the decisive parameters. The interesting connection between both concepts is that stock returns reflect the objectives of the financial market. On this basis, GDEA and the ideas of Edirisinghe and Zhang (2007, 2010) may be integrated.

https://doi.org/10.24355/dbbs.084-201902081010-0

References

Agrell, P.J. and Steuer, R.E. (2000), "ACADEA: a decision support system for faculty performance reviews", *Journal of Multi-Criteria Decision Analysis*, Vol. 9 No. 5, pp. 191–204.

Afsharian, M. and Ahn, H. (2014), "The Luenberger indicator and directions of measurement – a bottoms-up approach with an empirical illustration to German saving banks", *International Journal of Production Research*, Vol. 52 No. 20, pp. 6216–6233.

Ahn, H. and Le, M.H. (2014), "An insight into the specification of the input-output set for DEA-based bank efficiency measurement", *Management Review Quarterly*, Vol. 64 No. 1, pp. 3–37.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984), "Some models for estimating technical and scale inefficiency in data envelopment analysis", *Management Science*, Vol. 30 No. 6, pp. 1078–92.

Barnum, D.T., Shields, K.L., Walton, S.M. and Schumock, G.T. (2011), "Improving the efficiency of distributive and clinical services in hospital pharmacy", *Journal of Medical Systems*, Vol. 35 No. 1, pp. 59–70.

Başkaya, S. and Klumpp, M. (2014), "International data envelopment analysis in higher education: how do institutional factors influence university efficiency?", *Journal of Business and Economics*, Vol. 5 No. 11, pp. 2085–90.

Belton, V. and Stewart, T.J. (1999), "DEA and MCDA: competing or complementary approaches?", in Meskens, N. and Roubens, M. (Eds.), *Advances in decision analysis*, Kluwer Academic Publishers, Dordrecht et al., pp. 87–104.

Charnes, A., Cooper, W.W. and Rhodes, E. (1978), "Measuring the efficiency of decision making units", *European Journal of Operational Research*, Vol. 2 No. 2, pp. 429–44.

Cook W.D., Green, R.H. and Zhu, J. (2006), "Dual-role factors in data envelopment analysis, *IIE Transactions*, Vol. 38 No. 2, pp. 105–15.

Cook, W.D. and Seiford, L.M. (2009), "Data envelopment analysis (DEA): thirty years on", *European Journal of Operational Research*, Vol. 192 No. 1, pp. 1–17.

Dyckhoff, H. and Ahn, H. (2010), "Generalised DEA models for performance analysis" (Verallgemeinerte DEA-Modelle zur Performanceanalyse), *Journal of Business Economics*, Vol. 80 No. 12, pp. 1249–76.

Dyckhoff, H. and Allen, K. (2001), "Measuring ecological efficiency with data envelopment analysis (DEA)", *European Journal of Operational Research,* Vol. 132 No. 2, pp. 312–25.

Dyson, R.G., Allen, R., Camanho, A.S., Podinovski, V.V., Sarrico, C.S. and Shale, E.A. (2001), "Pitfalls and protocols in DEA", *European Journal of Operational Research*, Vol. 132 No. 2**,** pp. 245–59.

Eisenführ, F., Weber, M. and Langer, T. (2010), *Rational decision making*, Springer, Berlin et al.

Emrouznejad, A. and De Witte, K. (2010), "COOPER-framework: a unified process for non-parametric projects", *European Journal of Operational Research*, Vol. 207 No. 3, pp. 1573–86.

Edirisinghe, N.C.P. and Zhang, X. (2007), "Generalized DEA model of fundamental analysis and its application to portfolio optimization", *Journal of Banking and Finance*, Vol. 31 No. 11, pp. 3311–35.

Edirisinghe, N.C.P. and Zhang, X. (2010), "Input/output selection in DEA under expert information, with application to financial markets", *European Journal of Operational Research*, Vol. 207 No. 3, pp. 1669–78.

Fandel, G. (2007), "On the performance of universities in North Rhine-Westphalia, Germany: government's redistribution of funds judged using DEA efficiency measures", *European Journal of Operational Research*, Vol. 176 No. 1, pp. 521–33.

Feletto, E., Wilson, L.K. and Benrimoj, S.I. (2010), "Building capacity to implement cognitive pharmaceutical services: quantifying the needs of community pharmacies", *Research in Social and Administrative Pharmacy*, Vol. 6 No. 3, pp. 163–73.

Fethi, M.D. and Pasiouras, F. (2010), "Assessing bank efficiency and performance with operational research and artificial intelligence techniques: a survey", *European Journal of Operational Research*, Vol. 204 No. 2, pp. 189–98.

https://doi.org/10.24355/dbbs.084-201902081010-0

Golany, B. and Roll, Y. (1989), "An application procedure for DEA", *Omega*, Vol. 17 No. 3, pp. 237–50.

Johnes, J. and Johnes, G. (1995), "Research funding and performance in U.K. university departments of economics: a frontier analysis", *Economics of Education Review*, Vol. 14 No. 3, pp. 301–14.

Kaplan, R.S. and Norton, D.P. (2003), "Keeping your balance with customers", available at: http://hbswk.hbs.edu/item/3588.html (accessed 20 July, 2015).

Keeney, R.L. (1992), *Value-focused thinking: a path to creative decisionmaking*, Harvard University Press, Cambridge/London.

Keeney, R.L. and Raiffa, H. (1993), *Decisions with multiple objectives: preferences and value tradeoffs*, 2nd ed., Cambridge University Press, Cambridge.

Lozano, S., Gutiérrez, E. and Moreno, P. (2013), "Network DEA approach to airports performance assessment considering undesirable outputs", *Applied Mathematical Modelling*, Vol. 37 No. 4, pp. 1665–76.

Paradi, J.C. and Zhu, H. (2013), "A survey on bank branch efficiency and performance research with data envelopment Analysis", *Omega,* Vol. 41 No. 1, pp. 61–79.

Pathomsiri, S., Haghani, A., Dresner, M. and Windle, R.J. (2008), "Impact of undesirable outputs on the productivity of US airports", *Transportation Research Part E*, Vol. 44 No. 2, pp. 235–59.

Ramli, N.A. and Munisamy, S. (2013), "Modeling undesirable factors in efficiency measurement using data envelopment analysis: a review", *Journal of Sustainability Science and Management*, Vol. 8 No. 1, pp. 126–35.

Rickards, R.C. (2003), "Setting benchmarks and evaluating balanced scorecards with data envelopment analysis", *Benchmarking: An International Journal*, Vol. 10 No. 3, pp. 226–245.

Rousseau, S. and Rousseau, R. (1997), "Data envelopment analysis as a tool for constructing scientometric indicators", *Scientometrics*, Vol. 40 No. 1, pp. 45–56.

Schumock, G.T., Shields, K.L., Walton, S.M. and Barnum, D.T. (2009), "Data envelopment analysis: a method for comparing hospital pharmacy productivity", *American Journal of Health-System Pharmacy*, Vol. 66 No. 18, pp. 1660–5.

https://doi.org/10.24355/dbbs.084-201902081010-0

Seiford, L.M. and Zhu J. (2002), "Modeling undesirable factors in efficiency evaluation", *European Journal of Operational Research*, Vol. 142 No. 1, pp. 16–20.

Shutt, J.A. (2003), "Balancing the health care scorecard", in: *Managed Care*, Vol. 12, pp. 43–4.

Thanassoulis, E. (2001), *Introduction to the theory and application of data envelopment analysis: a foundation text with integrated software*, Kluwer Academic Publishers, Norwell.

Wagner, J.M. and Shimshak, D.G. (2007), "Stepwise selection of variables in data envelopment analysis: procedures and managerial perspectives", *European Journal of Operational Research*, Vol. 180 No. 1, pp. 57–67.

White, L. and Klinner C. (2012), "Service quality in community pharmacy: an exploration of determinants", *Research in Social and Administrative Pharmacy*, Vol. 8 No.2 , pp. 122–132.

Yu, M.-M., Hsu, S.-H., Chang, C.-C. and Lee, D.-H. (2008), "Productivity growth of Taiwan's major domestic airports in the presence of aircraft noise", *Transportation Research Part E*, Vol. 44 No. 3, pp. 543–54.

https://doi.org/10.24355/dbbs.084-201902081010-0

Table I. Characterization of attributes as performance criteria to measure the achievement of objectives

kind of attribute	**description**	**example**	**remarks**
natural attribute	arises directly from the objective under consideration and provides an unambiguous measurement	the *cost of service personnel* and the *cost of expired drugs* may be classified as natural attributes to measure the objective *cost of service*	the prerequisite is that these attributes are really equivalent to the cost of service
proxy attribute	an indicator or a means for the achievement of an objective	the *number of complaints about waiting time in the store* represents an indicator for the objective *waiting time in the store*	data availability may be the reason why not the waiting time of the customers itself is measured (as natural attribute), but the respective complaints
artificial attribute	a constructed combination of criteria relevant for the objective	the *telephone hotline hours for health counselling services* and the *number of health counselling services in the store* may together reflect the objective *customer advisory service concerning drugs*	artificial attributes can be seen as a combination of several proxy attributes; principally, there are endless options for such a combination